Let's derive more formulas for the derivative:

Recall: Inverse Function Theorem

If $f \circ g(x) = x$ i.e. $f(g(x)) = x$

Then $$(g(x))' = \frac{1}{f'(g(x))}$$

Example: Derive the formula for the derivative of $\arcsin(x)$.

Proof: $\sin(\arcsin(x)) = x$

By Inv Fun. Thm, $$(\arcsin(x))' = \boxed{\frac{1}{\cos(\arcsin(x))}}$$

$(\sin(x))' = \cos(x)$.

Side Calculation: What is $\cos(\arcsin(x))$

$\sin\theta = \frac{opp}{hyp} = \frac{x}{1}$

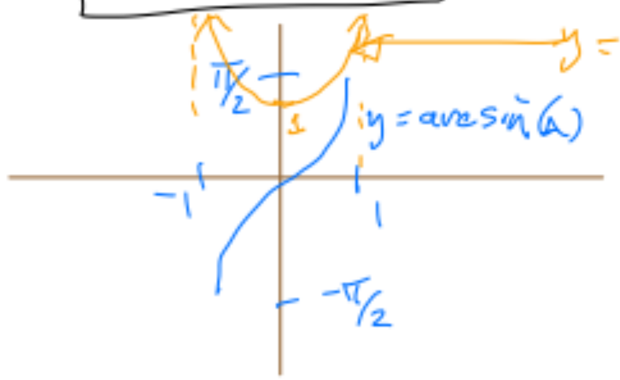
$\arcsin(x) = \theta$

$\Rightarrow \cos\theta = \frac{adj}{hyp} = \frac{bubba}{1} = \sqrt{1-x^2}$

$bubba = \sqrt{1-x^2}$

$$\Rightarrow \cos(\theta) = \boxed{\cos(\arcsin(x)) = \sqrt{1-x^2}}$$

$$\therefore \boxed{(\arcsin(x))'} = \frac{1}{\cos(\arcsin(x))} = \boxed{\frac{1}{\sqrt{1-x^2}}}$$



$y = \frac{1}{\sqrt{1-x^2}}$ = slope of arcsin(x)

$$(\arccos(x))' = \left(\frac{\pi}{2} - \arcsin(x)\right)'$$

$$= \boxed{\frac{-1}{\sqrt{1-x^2}}}$$

Example: Derive the formula for $(\arctan(x))'$.

Solution: $\tan(\arctan(x)) = x$

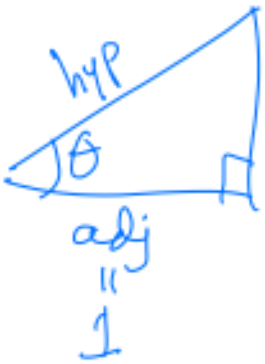
Inv. Fun Thm $\Rightarrow (\arctan(x))' = \boxed{\frac{1}{\sec^2(\arctan(x))}}$

$(\tan(x))' = \sec^2(x)$

Side Calculation: what is $\sec^2(\arctan(x))$

$$\theta = \arctan(x)$$

$$\tan\theta = x = \frac{x}{1} = \frac{\text{opp}}{\text{adj}}$$



$\text{opp} = x \Rightarrow \text{hyp} = \sqrt{1+x^2}$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{\text{hyp}}{\text{adj}} = \frac{\sqrt{1+x^2}}{1}$$

$$= \sqrt{1+x^2}$$

$$\Rightarrow \sec^2(\arctan(x)) = \sec^2(\theta)$$

$$= 1+x^2$$

$$\Rightarrow \boxed{(\arctan(x))' = \frac{1}{1+x^2}}$$

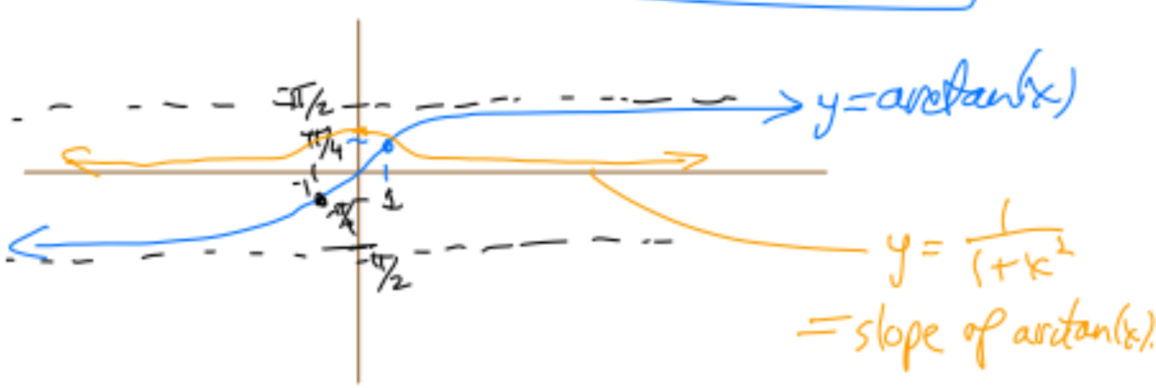


Table of Fast Derivatives

Linearity $(cf(x))' = cf'(x)$, $\left(f(x) \pm g(x)\right)' = f'(x) \pm g'(x)$

$\left(f(x)g(x)\right)' = f'(x)g(x) + f(x)g'(x)$ Product Rule

$\left(\frac{f(x)}{g(x)}\right)' = \frac{\text{Ho dHi} - \text{Hi dHo}}{\text{Ho Ho}} = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$ Quotient Rule

$\left(f(g(x))\right)' = f'(g(x)) \cdot g'(x)$ Chain Rule

If $f(g(x)) = x$, then $g'(x) = \frac{1}{f'(g(x))}$ Inverse Function Thm.

$(x^n)' = nx^{n-1}$ Power Rule

$(e^x)' = e^x \implies (\ln(x))' = \frac{1}{x}$

$(a^x)' = \ln(a)a^x$

$(\sin(x))' = \cos(x)$

$(\cos(x))' = -\sin(x)$

$(\tan(x))' = \frac{1}{\cos^2(x)} = \sec^2(x)$

$(\sec(x))' = \frac{\sin(x)}{\cos^2(x)} = \sec(x)\tan(x).$

$(\csc(x))' = -\csc(x)\cot(x) = -\frac{\cos(x)}{\sin^2(x)}.$

$(\cot(x))' = \frac{-1}{\sin^2(x)} = -\csc^2(x)$

$(\arcsin(x))' = \frac{1}{\sqrt{1-x^2}}$

$(\arccos(x))' = \frac{-1}{\sqrt{1-x^2}}$

$(\arctan(x))' = \frac{1}{1+x^2}$

$(\text{arccot}(x))' = \frac{-1}{1+x^2}$

$$(\operatorname{arcsec}(x))' = \frac{1}{|x|\sqrt{x^2-1}} \quad (\operatorname{arccsc}(x))' = \frac{-1}{|x|\sqrt{x^2-1}}$$